Math 10A
Quiz 6; Tuesday, 7/17/2018
Time: 3 PM
Instructor: Roy Zhao **Name:** ____________________

Circle True or False. (1 point for correct answer, 0 if incorrect)

1. True **FALSE** We can tell whether the midpoint rule approximating is an over or underestimating by looking at the first derivative.

   Solution: We need to look at the second derivative.

2. **TRUE** False Using Simpson's method will always give the exact answer when integrating a quadratic equation.

   Solution: For a quadratic equation, $K_4 = 0$ so there is no error.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (4 points) Approximate $\int_{-\pi}^{\pi} \cos(x)dx$ using the Trapezoid rule with $n = 4$ trapezoids. (Simplify your answer)

   Solution: First we find $\Delta x = \frac{\pi-(-\pi)}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$. Using the formula, the area using Trapezoid rule is:

   $$\frac{\pi/2}{2}(\cos(-\pi)+2\cos(-\pi/2)+2\cos(0)+2\cos(\pi/2)+\cos(\pi)) = \frac{\pi}{4}(-1+0+2+0+-1) = 0.$$

   (b) (3 points) What is the maximum error in the previous estimation?

   Solution: The maximum error is $\dfrac{K_2(b-a)^3}{12n^2}$. The second derivative is $-\cos(x)$ and the maximum of $|f''(x)| = |-\cos(x)|$ is 1 so $K_2 3 = 1$. So the error is

   $$\frac{1 \cdot (\pi - (-\pi))^3}{12 \cdot 4^2} = \frac{8\pi^3}{12 \cdot 16} = \frac{\pi^3}{24}.$$

(c) (3 points) What is the smallest number of intervals n you need to use in order to guarantee that the trapezoid approximation of $\int_0^1 \frac{x^3}{6} dx$ is within $\frac{1}{12 \cdot 101}$.

Solution: First we need to calculate $K_2 = \max_{[0,1]} |(\frac{x^3}{6})''|$. The first derivative is $\frac{x^2}{2}$ and the second is x so the maximum of the absolute value is at $x = 1$ since the function x is always increasing. Therefore, $K_2 = 1$. Thus, we have that

$$\frac{1}{12 \cdot 101} = \frac{1(1-0)^3}{12n^2} \implies n^2 = \frac{12 \cdot 101}{12} = 101.$$

Therefore $n = \sqrt{101}$. But, we want the smallest number of intervals and so we need to take the ceiling. The ceiling of $\sqrt{101}$ is 11 so $N = 11$.